Face Image Retrieval System using Discrete Orthogonal Moments

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Abstract. Image Retrieval from large databases is an embryonic application predominantly in medical and forensic departments. Face image retrieval is quiet a thought-provoking task since face images can vary noticeably in terms of facial expressions, lighting conditions etc. In feature based image retrieval methods, the accuracy depends on the discrimination power of the features. In this work, orthogonal moments were employed as features for the retrieval task. Due to the orthogonal property, these moments are inherently non-redundant revealing good image representation capability. Racah moments defined in a non-uniform lattice are evidenced to be better than other orthogonal moments in terms of reconstruction error. Face image retrieval using Dual Hahn moment, Racah moment and Tchebicef moment features has been extensively experimented with YALE face database and FERET Database. The results divulge the effectiveness of orthogonal moment descriptors.

Keywords: Feature selection, Image Retrieval, Moment feature, Racah Moment.

1. Introduction

The image retrieval is to find effectively the image data interested by the users from a large collection of image databases [1, 2]. In this paper, the focus is placed on building a competent and precise image retrieval system. To date, several feature based image retrieval systems have been proposed [3-12]. Automatic face analysis which includes, face detection, faces recognition, and facial expression recognition has become a very active topic in computer vision research. Different holistic methods such as Principal Component Analysis(PCA) [13], Linear Discriminant Analysis (LDA) [15], and 2D PCA [14] have been studied widely but lately also local descriptors have gained attention due to their robustness to challenges such as pose and illumination changes. One of the first face descriptors based on information extracted from local regions is the eigen features method proposed by Pentland et al. [16]. This paper proposes to employ Orthogonal moment to extract moment features and obtain higher retrieval rate. The remainder of the paper is organized as follows. Section II exemplifies the Extraction of Orthogonal Moment features. Section III describes the Image Retrieval Algorithm. Section IV details the experimental evaluation and Section V presents the Conclusions.

2. Extraction of Orthogonal Moment Features

Moments have been extensively used in image processing, pattern recognition and Computer vision [17-20]. Teague proposed continuous orthogonal polynomials as the basis functions to calculate continuous moments [21]. These orthogonal polynomials include Zernike polynomials, Legendre polynomials, pseudo-Zernike and Tchebichef polynomials. Since Hu (1962) introduced moment invariants, moments and functions of moments due to their ability to represent global features of an image have found wide applications in the fields of image processing and pattern recognition [22,23], image indexing [24] robust line fitting [25], and image recognition [26]. Among the different types of moments, the Cartesian geometric moments are most extensively used. It was shown [27] that the discrete orthogonal moments perform better than the conventional continuous orthogonal moments in terms of image representation capability. In this paper, we present a new set of discrete orthogonal polynomials, Dual Hahn and Racah polynomials, which
are orthogonal on a non-uniform lattice. The dual Hahn and Racah polynomials are scaled, to ensure that all the computed moments have equal weights, and are used to define a new type of discrete orthogonal moments known as Dual Hahn and Racah moments respectively. It is significant that although the Dual Hahn and Racah polynomials are orthogonal on a non-uniform lattice, the discrete Racah moments [28] and Dual Hahn moments [29] defined in this paper are applied to uniform pixel grid image. The difference between these moments and the discrete moments based on the polynomials that are orthogonal on uniform lattice is that the latter is directly defined on the image grid but, for the former, we should introduce an intermediate, non-uniform lattice, x(s) = s(s + 1).

### 2.1. Computation of Racah Moments

The Classical Racah polynomials $U_n^{(\alpha, \beta)}$ [28] (s,a,b), n = 0, 1, ..., L-1, defined on a non uniform lattice x(s) = s(s + 1) and the weighting function $\rho(s)$ is given by

$$\rho(s) = \frac{\Gamma(a+s+1)\Gamma(b+\alpha-s)\Gamma(b+s+1)}{\Gamma(a+b+s+2)\Gamma(b+\alpha-s)}$$

where the parameters a, b, $\alpha$, and $\beta$ are restricted to $-1/2 < a < b$, $-1 < b < 2a + 1$, $b = a + N$.

The $n$th order Racah polynomials are defined as

$$U_n^{(\alpha, \beta)}(s,a,b) = \frac{1}{n!} (a-b+1)_n (f+1)_n (a+b+\alpha+1)_n x \binom{n}{a}^n (a-b+1)_n (f+1)_n (a+b+\alpha+1)_n x \binom{n}{a}^n (a-b+1)_n (f+1)_n (a+b+\alpha+1)_n x \binom{n}{a}^n$$

N = 0, 1, ..., L-1, s = a, a+1, ..., b-1, Where $(u)_k$ is the Pochhammer symbol defined as

$$(u)_k = u(u+1)…(u+k-1) = \frac{\Gamma(u+k)}{\Gamma(u)}$$

And $\binom{n}{a}^n$ is the generalized hypergeometric function given by

$$\binom{n}{a}^n = \sum_{k=0}^{\infty} \frac{(a)_k (a+1)_k … (a+n-1)_k}{(b1)_k (b2)_k … (bn)_k} \frac{z^k}{k!}$$

to avoid numerical instability in polynomial computation, the racah polynomials are scaled by utilizing the square norm and the weighting function. The set of weighted racah polynomials is defined as

$$\tilde{U}_n^{(\alpha, \beta)}(s,a,b) = \sqrt{\rho(s)} U_n^{(\alpha, \beta)}(s,a,b)$$

The Racah moments are a set of moments derived by using the weighted Racah polynomials. Given a uniform pixel of lattices image f(s,t) with size N x N, the $(n+m)$th order Racah moment is defined as

$$\tilde{U}_{nm} = \frac{1}{d_s^2} \sum_{s=a}^{b-1} \sum_{t=a}^{b-1} \tilde{U}_n^{(\alpha, \beta)}(s,a,b) \tilde{U}_n^{(\alpha, \beta)}(t,a,b) f(s,t)$$

The orthogonality property of the Racah polynomials helps in expressing the image intensity functions $f(s,t)$ in terms of its Racah moments.

### 2.2. Computation of Dual Hahn Moments

The Classical dual Hahn polynomials $w_n^{(c)}$ [30] (s,a,b), n = 0, 1, ..., N-1, defined on a non uniform lattice x(s) = s(s + 1) and the weighting function $\rho(s)$ is given by

$$\rho(s) = \frac{\Gamma(a+s+1)\Gamma(c+s+1)}{\Gamma(s-a+1)\Gamma(b-s)\Gamma(b+s+1)\Gamma(s-c+1)}$$

where the parameters a, b, and c are restricted to $-1/2 < a < b$, $|c| < 1+a$, $b = a + N$.

The $n$th order dual Hahn polynomials are defined as
\[
\begin{align*}
\hat{w}_n^{(c)}(s,a,b) &= \frac{(a-b+1)(a+c+1)}{n!} {}_3F_2(-n,a-s,a+s+1; a-b+1,a+c+1;1) \\
\end{align*}
\]  
(9)

Where \((u)_k\) is the Pochhammer symbol defined as
\[
(u)_k = u(u+1)\ldots(u+k-1) = \frac{\Gamma(u+k)}{\Gamma(u)}
\]  
(10)

And \(_3F_2(.)\) is the generalized hypergeometric function given by
\[
{}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k(a_2)_k(a_3)_k}{(b_1)_k(b_2)_k} \frac{z^k}{k!}
\]  
(11)

to avoid numerical instability in polynomial computation, the dual Hahn polynomials are scaled by utilizing the square norm and the weighting function. The set of weighted dual Hahn polynomials is defined as
\[
\hat{w}_n^{(c)}(s,a,b) = w_n^{(c)}(s,a,b) \sqrt{\frac{\rho(S)}{d^2}} [\Delta x(s - \frac{1}{2})] \quad n=0,1,\ldots,N-1
\]  
(12)

Where
\[
d^2 = \frac{\Gamma(a+c+n+1)}{n!(b-a-n-1)!\Gamma(b-c-n)}, n=0,1,\ldots,N-1
\]  
(13)

The dual Hahn moments are a set of moments derived by using the weighted dual Hahn polynomials. Given a uniform pixel of lattices image \(f(s,t)\) with size \(N \times N\), the \((n+m)\)th order dual Hahn moment is defined as
\[
W_{nm} = \sum_{s=0}^{N-1} \sum_{t=0}^{N-1} \hat{w}_n^{(c)}(s,a,b) \hat{w}_m^{(c)}(t,a,b)f(s,t) \quad n,m=0,1,\ldots,N-1
\]  
(14)

The orthogonality property of the dual Hahn polynomials helps in expressing the image intensity functions \(f(s,t)\) in terms of its dual Hahn moments.

### 2.3. Computation of Tchebichef Moments

The \((p+q)\)th order Tchebichef moment is defined as
\[
T_{pq} = \frac{1}{\rho(p,N)\rho(q,N)} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} t_p(x)t_q(y)f(x,y), \quad p,q = 0,1,2,\ldots,N-1
\]  
(15)

Where
\[
\rho(p,N) = \frac{N(1 - \frac{1}{N^2})\frac{1}{N^2}(1 - \frac{2^2}{N^2})\ldots(1 - \frac{p^2}{N^2})}{2p+1}
\]  
(16)

is the squared norm and \(t_p(x)\) is the scaled Tchebichef polynomial of order \(p\).

The orthogonal Tchebichef polynomials are defined by the recursive relation
\[
\hat{t}_p(x) = \alpha(2x+1-N)\hat{t}_{p-1}(x) + \beta\hat{t}_{p-2}(x), \quad p=0,1,2,\ldots,N-2; \quad x=0,1,2,\ldots,N-1.
\]  
(17)

Where
\[
\alpha = \frac{\sqrt{4p^2 - 1}}{p\sqrt{N^2 - p^2}}
\]  
(18)

\[
\beta = -\frac{(p-1)\sqrt{2p+1}\sqrt{N^2 - (p-1)^2}}{p\sqrt{2p-3}\sqrt{N^2 - p^2}}
\]  
(19)

The initial conditions of the above recurrence relation are
\[
\hat{t}_0(x) = N^{-\frac{1}{2}}
\]  
(20)
\[
\hat{t}_1(x) = \frac{\sqrt{3}(2x+1-N)}{\sqrt{N(N^2 - 1)}}
\]  
(21)

The discrete orthogonal polynomials defined as above satisfy the following condition for all \(p\):
\[ \rho(p, N) = \sum_{i=0}^{N-1} \left\{ \hat{t}_p(i) \right\}^2 = 1.0 \]  

The Tchebichef polynomials in equation 3 can be renormalized to minimize propagation of any numerical errors through the recurrence relation

\[ \hat{t}_p(x) = \frac{\hat{t}_p(x)}{\sqrt{\sum_{i=0}^{N-1} (\hat{t}_p(x))^2}} \]  

It is observed that the reconstruction accuracy improves considerably by the renormalization of the orthogonal Tchebichef moments.

3. Image Retrieval Algorithm

The value of parameter \( a \) influences the selection of exact order moment in image retrieval, i.e we have analytically set \( a=\alpha \) and \( b=\alpha+N \). Several cases with different value of parameter are tested and the patterns are reconstructed with a moment order from 10 to 40. It is also observed that, the reconstructed images with \( a=\alpha=9 \) and \( b=49 \) are better. The accuracy of reconstructed image is found by calculating the reconstruction error. Beyond moment order 25, the reduction in reconstruction error with different value of parameters is negligible. Hence Dual Hahn moment, Racah moments and Tchebichef moment upto order 25 are selected as features for the face image retrieval task.

4. Experimental Study

The efficiency of this image retrieval system is tested on YALE and FERET image database. Results for the retrieval of the 10 most similar image from a query image is illustrated. The retrieval rate for the query image is measured by counting the number of images from the same category which are found in the top \( k \) matches. The retrieval rate is provided in Table 1.

<table>
<thead>
<tr>
<th>Images Database</th>
<th>Dual Hahn Moment Feature</th>
<th>Racah Moment Feature</th>
<th>Tchebichef Moment Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>FERET Database</td>
<td>97%</td>
<td>96%</td>
<td>91%</td>
</tr>
<tr>
<td>YALE Database</td>
<td>93%</td>
<td>94%</td>
<td>84%</td>
</tr>
</tbody>
</table>

5. Conclusion

This paper proposes a new approach for face recognition using discrete orthogonal moments. Two different databases have been used to evaluate the proposed method. The orthogonal moment features prove to be efficacious for the image retrieval task.

6. References


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